

Reading: Read Chapter 1 (except for the Matrix norms section).

Do the following exercises showing all work.

1. Consider the system

$$\begin{aligned}4x_1 & & - & x_3 & = & 200 \\9x_1 + x_2 & - & x_3 & = & 200 \\7x_1 - x_2 + 2x_3 & = & 200.\end{aligned}$$

- (a) (1 point) Write the augmented matrix corresponding to this system.
(b) (3 point) Reduce the augmented system in part (a) to echelon form.
(c) (2 point) Describe the set of solutions to the given system.
2. (6 points) Solve the following system of linear equations

$$\begin{aligned}x_1 + 2x_2 & = & 1 \\-x_1 - 4x_2 + x_3 & = & 2 \\2x_2 + x_3 & = & 0.\end{aligned}$$

3. Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}.$$

- (a) (3 points) Find the eigenvalues of the matrix A . Is any eigenvalue repeated?
(b) (4 points) Find three eigenvectors u_1, u_2, u_3 of A that are orthonormal.
(c) (1 point) State a spectral (eigenvalue) decomposition of A .
4. Find the local and global minimizers and maximizers of the following functions.
- (a) (2 points) $f(x) = x^2 + 2x$
(b) (2 points) $f(x) = x^2 e^{-x^2}$
(c) (2 points) $f(x) = x^2 + \cos x$
(d) (2 points) $f(x) = x^3 - x$
5. Compute the gradient $\nabla f(x)$ of the following functions.
- (a) (2 points) $f(x) = x_1^3 + x_2^3 - 3x_1 - 15x_2 + 25$ on \mathbb{R}^2
(b) (2 points) $f(x) = x_1^2 + x_2^2 - \sin(x_1 x_2)$ on \mathbb{R}^2
(c) (2 points) $f(x) = \|x\|^2 = \sum_{j=1}^n x_j^2$ on \mathbb{R}^n

(d) (2 points) $f(x) = e^{\|x\|^2}$ on \mathbb{R}^n

(e) (2 points) $f(x) = x_1 x_2 x_3 \cdots x_n$ on \mathbb{R}^n

(f) (2 points) $f(x) = -\log(x_1 x_2 x_3 \cdots x_n)$ on the set $\{x \in \mathbb{R}^n : x_i > 0 \text{ for all } i = 1, \dots, n\}$

6. (12 points) Compute the Hessian $\nabla^2 f(x)$ of the functions given in problem 5 above.